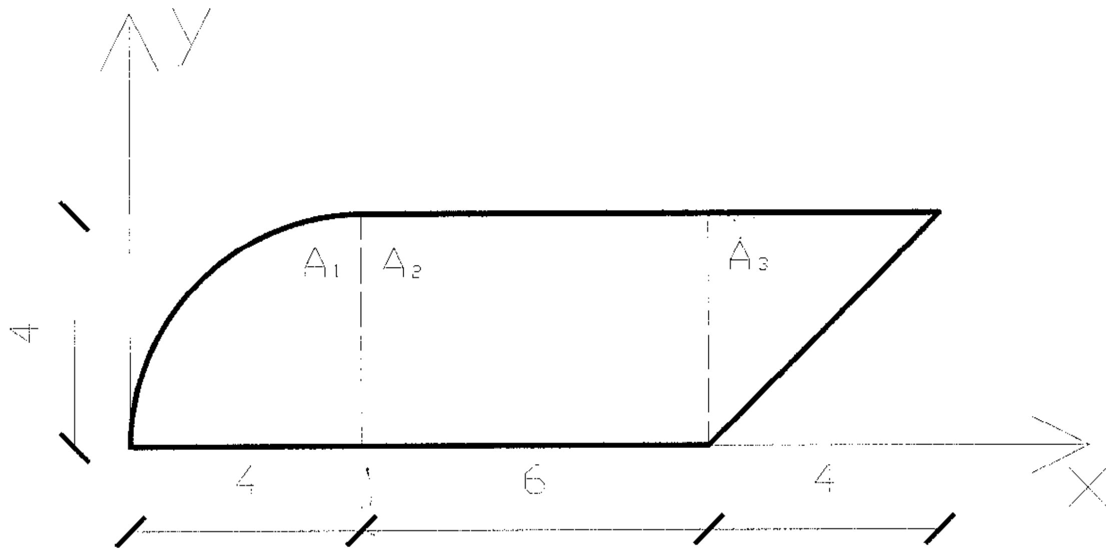
Zadanie nr 1.

Dla przekroju pokazanego na rysunku wyznaczyć środkową elipsę bezwładności.



Pole przekroju

$A_c = A_1 + A_2 + A_3$
$A_c = 0{,}25\pi R^2 + 4cm \cdot 6cm + 0{,}5 \cdot 4cm \cdot 4cm = 44{,}5664\ cm^2$

Moment statyczny przekroju

$$S_x = \Sigma A_i \cdot y_i$$
$$S_x = 12{,}5664cm^2 \cdot 1{,}7cm + 24cm^2 \cdot 2cm + 8cm^2 \cdot 2{,}6667cm = 90{,}69621cm^3$$

$$S_y = \Sigma A_i \cdot x_i$$
$$S_y = 12{,}5664cm^2 \cdot 2{,}3cm + 24cm^2 \cdot 7cm + 8cm^2 \cdot 11{,}3333cm = 287{,}56939cm^3$$

Środek ciężkości przekroju

$$x_o = \frac{S_y}{A_c}$$
$$x_o = \frac{287{,}56939cm^3}{44{,}5664cm^2} = 6{,}4526cm$$

$$y_o = \frac{S_x}{A_c}$$

$$y_o = \frac{90{,}69621cm^3}{44{,}5664cm^2} = 2{,}0351cm$$

Momenty bezwładności

$$Jx_o = 0{,}0549 \cdot 4^4 + \left[(1{,}7 - 2{,}0351)^2 \cdot 12{,}5664\right] + \frac{6 \cdot 4^3}{12} + \left[(2 - 2{,}0351)^2 \cdot 24\right] + \frac{4 \cdot 4^4}{36} + \left[(2{,}6667 - 2{,}0351)^2 \cdot 8\right] =$$
$$= 57{,}7972cm^4$$

$$Jy_o = 0{,}0546 \cdot 4^4 + \left[(2{,}3 - 6{,}4526)^2 \cdot 12{,}5664\right] + \frac{6^3 \cdot 4}{12} + \left[(7 - 6{,}4526)^2 \cdot 24\right] + \frac{4^3 \cdot 4}{36} + \left[(11{,}3333 - 6{,}4526)^2 \cdot 8\right] =$$
$$= 507{,}6256cm^4$$

$$Jx_oy_o = 0{,}0165 \cdot 4^4 + \left[(1{,}7 - 2{,}0351) \cdot (2{,}3 - 6{,}4526) \cdot 12{,}5664\right] - 0 + \left[(2 - 2{,}0351) \cdot (7 - 6{,}4526) \cdot 24\right] +$$
$$+ \frac{4^2 \cdot 4^2}{72} + \left[(2{,}6667 - 2{,}0351) \cdot (11{,}3333 - 6{,}4526) \cdot 8\right] = 45{,}3149cm^4$$

Wyznaczenie położenia osi głównych

$$tg2\alpha_0 = \frac{-2 \cdot Jx_oy_o}{Jx_o - Jy_o}$$

$$tg2\alpha_0 = \frac{-2 \cdot 45{,}3149cm^4}{57{,}7972cm^4 - 507{,}6256cm^4} = 0{,}201476$$

$$2\alpha_0 = 11{,}391225^\circ$$

$$\alpha_0 = 5{,}695613^\circ$$

Wyznaczenie momentów głównych względem osi głównych

$$J_{max} = \frac{Jx_o + Jy_o}{2} + \frac{1}{2} \cdot \sqrt{(Jx_o - Jy_o)^2 + 4 \cdot Jx_oy_o^{\ 2}}$$

$$J_{max} = \frac{57{,}7972cm^4 + 507{,}6256cm^4}{2} + \frac{1}{2} \cdot \sqrt{\left(57{,}7972cm^4 - 507{,}6256cm^4\right)^2 + 4 \cdot \left(45{,}3149cm^4\right)^2} = 512{,}145132cm^4$$

$$J_{min} = \frac{Jx_o + Jy_o}{2} - \frac{1}{2} \cdot \sqrt{(Jx_o - Jy_o)^2 + 4 \cdot Jx_oy_o^{\ 2}}$$

$$J_{max} = \frac{57{,}7972cm^4 + 507{,}6256cm^4}{2} - \frac{1}{2} \cdot \sqrt{\left(57{,}7972cm^4 - 507{,}6256cm^4\right)^2 + 4 \cdot \left(45{,}3149cm^4\right)^2} = 53{,}277668cm^4$$

Sprawdzenie

$$Jx_o + Jy_o = J_{\max} + J_{\min}$$
$$57{,}7972cm^4 + 507{,}6256cm^4 = 512{,}145132cm^4 + 53{,}277668cm^4$$
$$565{,}4228cm^4 = 565{,}4228cm^4$$

$$Jx_{gl}y_{gl} = 0$$
$$Jx_{gl}y_{gl} = \frac{Jx_o - Jy_o}{2} \cdot \sin 2\alpha_o + Jx_o y_o \cdot \cos 2\alpha_o$$
$$Jx_{gl}y_{gl} = \frac{57{,}7972cm^4 - 507{,}6256cm^4}{2} \cdot \sin 2 \cdot 5{,}695613 + 45{,}3149cm^4 \cdot \cos 2 \cdot 5{,}695613 = 0{,}00086cm^4$$

Określenie osi głównych

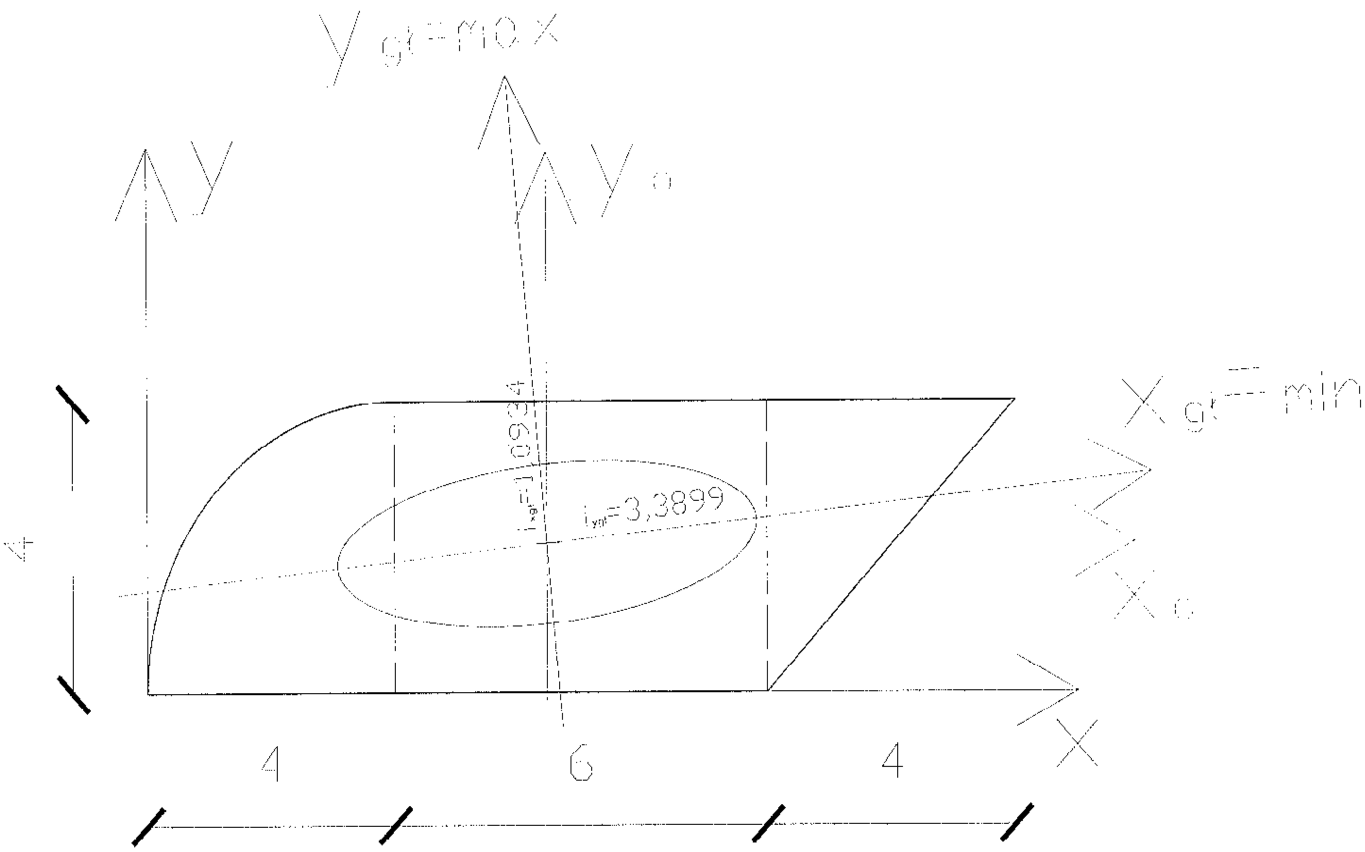
$$J_m = \frac{Jx_o + Jy_o}{2} + \frac{Jx_o - Jy_o}{2} \cdot \cos 2\alpha_o - Jx_o y_o \cdot \sin 2\alpha_o = x_{gl}$$
$$J_m = \frac{57{,}7972cm^4 + 507{,}6256cm^4}{2} + \frac{57{,}7972cm^4 - 507{,}6256cm^4}{2} \cdot \cos 2 \cdot 5{,}695613 -$$
$$- 45{,}3149cm^4 \cdot \sin 2 \cdot 5{,}695613 = 53{,}277668cm^4$$

$$J_n = \frac{Jx_o + Jy_o}{2} - \frac{Jx_o - Jy_o}{2} \cdot \cos 2\alpha_o + Jx_o y_o \cdot \sin 2\alpha_o = y_{gl}$$
$$J_m = \frac{57{,}7972cm^4 + 507{,}6256cm^4}{2} - \frac{57{,}7972cm^4 - 507{,}6256cm^4}{2} \cdot \cos 2 \cdot 5{,}695613 +$$
$$+ 45{,}3149cm^4 \cdot \sin 2 \cdot 5{,}6256 = 512{,}145132cm^4$$

Wyznaczenie promieni bezwładności elipsy

$$i_{xgl} = \sqrt{\frac{J_{\min}}{A_c}}$$
$$i_{xgl} = \sqrt{\frac{53{,}277668cm^4}{44{,}5664cm^2}} = 1{,}0934cm$$

$$i_{ygl} = \sqrt{\frac{J_{\max}}{A_c}}$$
$$i_{ygl} = \sqrt{\frac{512{,}145132cm^4}{44{,}5664m^2}} = 3{,}3899cm$$

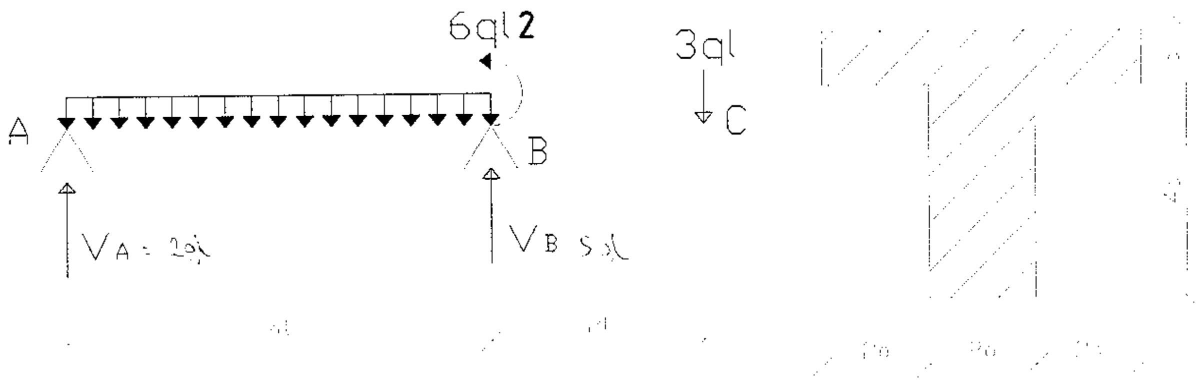


y_{gt}=max
y
y_o
X_{gt}=min
X_o
X
i_{xgt}=1,0934
i_{ygt}=3,3899
4
4
6
4

Zadanie nr 2.

Dla belki o danym przekroju jak na rysunku ($q=2kN/m$, $l=2m$)

- dobrać przekrój poprzeczny na maksymalny moment zginający i maksymalną siłę poprzeczną przyjmując $R=175$ MPa, $Rt=105$ MPa
- sporządzić wykresy ekstremalnych naprężeń maksymalnych i stycznych



$\Sigma X = 0$

$\underline{H_B = 0}$

$\Sigma M_B = 0$

$V_A \cdot 4l - q \cdot 4l \cdot 2l - 6ql^2 + 3ql \cdot 2l = 0$

$\underline{V_A = 2ql}$

$\Sigma Y = 0$

$2ql - 4ql + V_B - 3ql = 0$

$\underline{V_B = 5ql}$

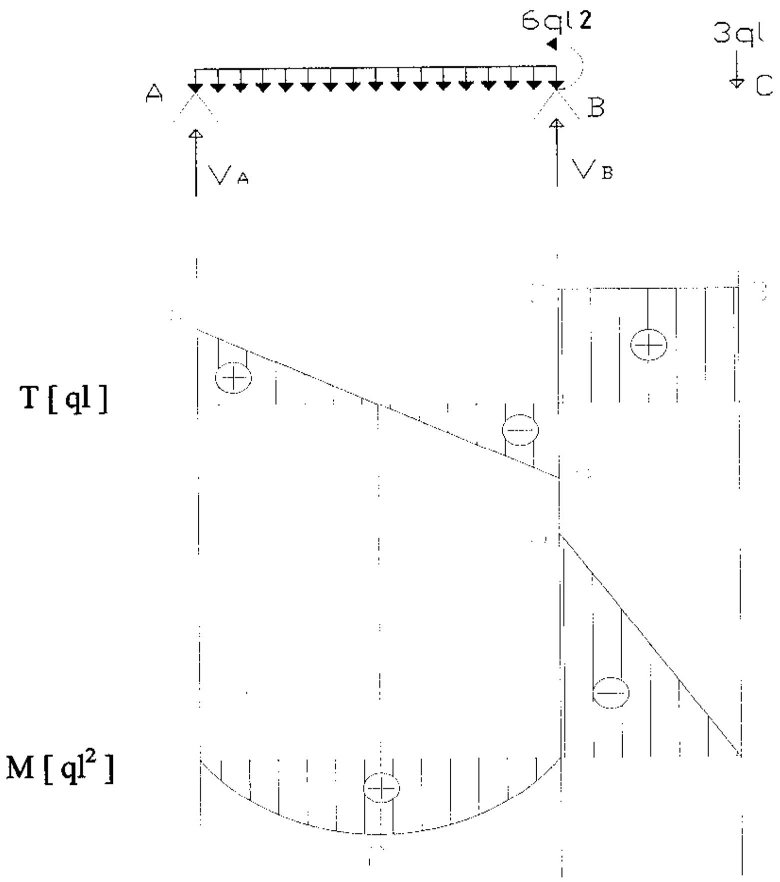
<u>Sprawdzenie:</u>

$\Sigma M_C = 0$

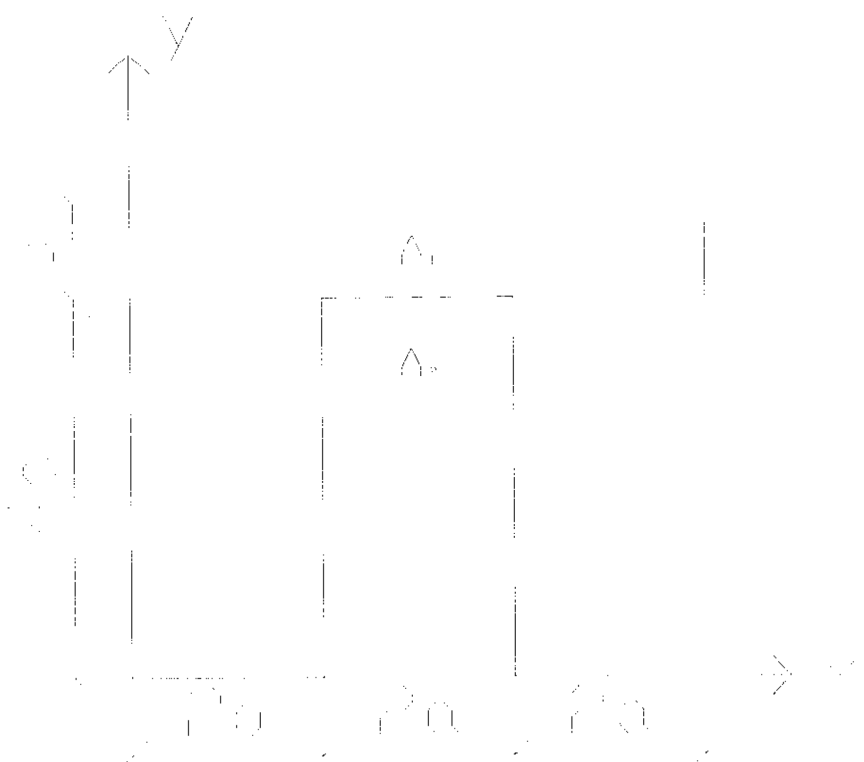
$V_A \cdot 6l - 4ql \cdot 4l - 6ql^2 + V_B \cdot 2l = 0$

$12ql^2 - 16ql^2 - 6ql^2 + 10ql^2 = 0$

$\underline{0 = 0}$



Mmax = $6ql^2$ = 48 kNm
Tmax = 3ql = 12 kN



$A_c = A_1 + A_2$
$A_c = 6a \cdot a + 4a \cdot 2a = 14a^2$

$$S_x = 6a^2 \cdot 4{,}5a + 8a^2 \cdot 2a = 43a^3$$

$$y_o = \frac{43a^3}{14a^2} = 3{,}07a$$

$$Jx_o = \frac{2a \cdot (4a)^3}{12} + \left[(2a - 3{,}07a)^2 \cdot 8a^2\right] + \frac{6a \cdot a^3}{12} + \left[(4{,}5a - 3{,}07a)^2 \cdot (6a)^2\right] = 32{,}6a^4$$

$$W_x^{\,g} = \frac{Jx_o}{y^g}$$

$$W_x^{\,g} = \frac{32{,}6a^4}{1{,}93a} = 16{,}9a^3$$

$$W_x^{\,d} = \frac{Jx_o}{y^d}$$

$$W_x^{\,d} = \frac{32{,}6a^4}{3{,}07a} = 10{,}62a^3$$

$$\delta_{\max} \le R$$

$$\frac{M_{\max}}{W_{\min}} \le R$$

$$\frac{48kNm}{10{,}63a^3} \le 175MPa$$

$$\frac{4800kNcm}{10{,}62a^3} \le 17{,}5\frac{kN}{cm^2}$$

$$a^3 \ge 25{,}83cm^3$$

$$a \ge 2{,}96cm$$

$$\tau_{\max} \le R_t$$

$$\frac{T_{\max} \cdot S\overline{x}}{b \cdot Jx_o} \le R_t$$

$$\frac{12kN \cdot (3{,}07a \cdot 2a \cdot 1{,}535a)}{2a \cdot 32{,}6a^4} \le 105MPa$$

$$\frac{113{,}10kNa^3}{65{,}2a^5} \le 10{,}5\frac{kN}{cm^2}$$

$$a^2 \ge 0{,}165cm^2$$

$$a \ge 0{,}41cm$$

Przyjęto a = 3,0cm.

$$\delta_{\max}{}^{g} = \frac{M_{\max}}{W_x{}^{g}} = \frac{48kNm}{16{,}9 \cdot 3{,}0^3} = \frac{4800kNcm}{456{,}3cm^3} = 10{,}52\frac{kN}{cm^2} = 105{,}20MPa$$

$$\delta_{\max}{}^{d} = \frac{M_{\max}}{W_x{}^{d}} = \frac{48kNm}{10{,}62 \cdot 3{,}0^3} = \frac{4800kNcm}{286{,}74cm^3} = 16{,}74\frac{kN}{cm^2} = 167{,}40MPa$$

$$\tau = \frac{T_{\max} \cdot S\overline{x}}{b \cdot Jx_o}$$

$$\tau_1 = 0$$

$$\tau_2 = \frac{12kN \cdot (6a \cdot a \cdot 1{,}43 \cdot a)}{6a \cdot 32{,}6a^4} = \frac{12kN \cdot (6 \cdot 3{,}0 \cdot 3{,}0 \cdot 1{,}43 \cdot 3{,}0)}{6 \cdot 3{,}0 \cdot 32{,}6 \cdot 3{,}0^4} = 0{,}0585\frac{kN}{cm^2} = 0{,}585MPa$$

$$\tau_3 = \frac{12kN \cdot (6a \cdot a \cdot 1{,}43a)}{2a \cdot 32{,}6a^4} = \frac{12kN \cdot (6 \cdot 3{,}0 \cdot 3{,}0 \cdot 1{,}43 \cdot 3{,}0)}{2 \cdot 3{,}0 \cdot 32{,}6 \cdot 3{,}0^4} = 0{,}1755\frac{kN}{cm^2} = 1{,}755MPa$$

$$\tau_{\max} = \tau_4 = \frac{12kN \cdot (3{,}07a \cdot 2a \cdot 1{,}535a)}{2a \cdot 32{,}6 \cdot a^4} = \frac{12kN \cdot (3{,}07 \cdot 3{,}0 \cdot 2 \cdot 3{,}0 \cdot 1{,}535 \cdot 3{,}0)}{2 \cdot 3{,}0 \cdot 32{,}6 \cdot 3{,}0^4} =$$

$$0{,}1927\frac{kN}{cm^2} = 1{,}927MPa$$

$$\tau_5 = 0$$