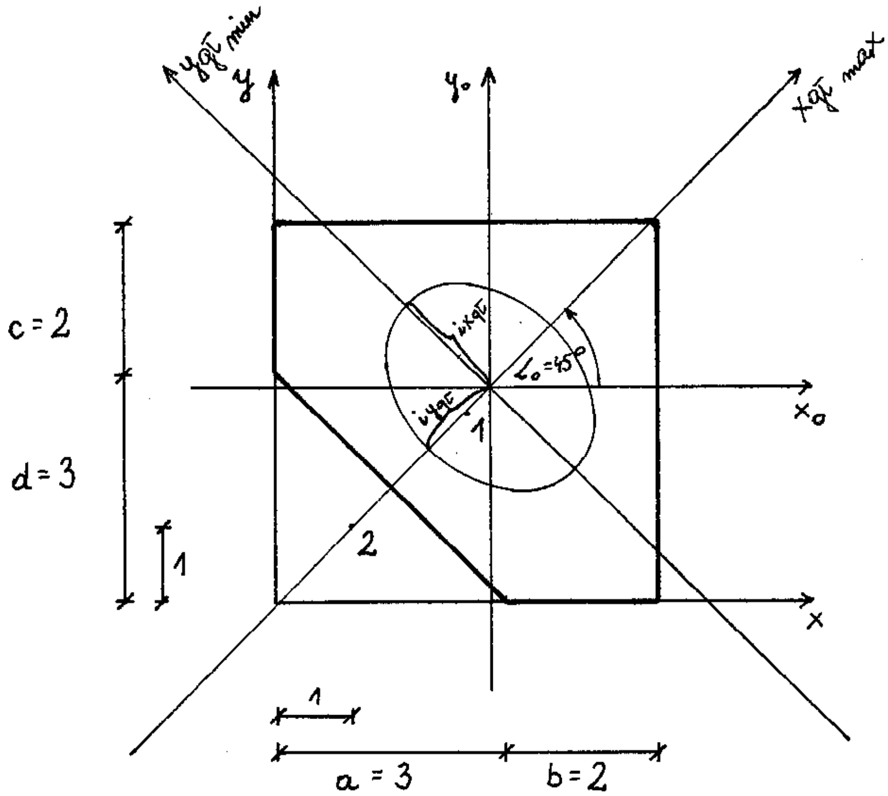


$$A = 5\ \text{cm} \cdot 5\ \text{cm} - \frac{1}{2} \cdot 3\ \text{cm} \cdot 3\ \text{cm} = 25\ \text{cm}^2 - 4{,}5\ \text{cm}^2 = 20{,}5\ \text{cm}^2$$

$$S_x = 25\ \text{cm}^2 \cdot 2{,}5\ \text{cm} - 4{,}5\ \text{cm}^2 \cdot 1\ \text{cm} = 62{,}5\ \text{cm}^3 - 4{,}5\ \text{cm}^3 = 58\ \text{cm}^3$$

$$S_y = 25\ \text{cm}^2 \cdot 2{,}5\ \text{cm} - 4{,}5\ \text{cm}^2 \cdot 1\ \text{cm} = 62{,}5\ \text{cm}^3 - 4{,}5\ \text{cm}^3 = 58\ \text{cm}^3$$

$$x_o = \frac{S_y}{A} = \frac{58\ \text{cm}^3}{20{,}5\ \text{cm}^2} = 2{,}83\ \text{cm}$$

$$y_o = \frac{S_x}{A} = \frac{58\ \text{cm}^3}{20{,}5\ \text{cm}^2} = 2{,}83\ \text{cm}$$

$$J_{xo} = \left[\frac{5\ \text{cm} \cdot (5\ \text{cm})^3}{12} + (2{,}5\ \text{cm} - 2{,}83\ \text{cm})^2 \cdot 25\ \text{cm}^2\right] - \left[\frac{3\ \text{cm} \cdot (3\ \text{cm})^3}{36} + (1\ \text{cm} - 2{,}83\ \text{cm})^2 \cdot 4{,}5\ \text{cm}^2\right]$$
$$= \left[52{,}08\ \text{cm}^4 + 2{,}72\ \text{cm}^4\right] - \left[2{,}25\ \text{cm}^4 + 15{,}07\ \text{cm}^4\right] = 54{,}80\ \text{cm}^4 - 17{,}32\ \text{cm}^4 = 37{,}48\ \text{cm}^4$$

$$J_{yo} = \left[\frac{(5\ \text{cm})^3 \cdot 5\ \text{cm}}{12} + (2{,}5\ \text{cm} - 2{,}83\ \text{cm})^2 \cdot 25\ \text{cm}^2\right] - \left[\frac{(3\ \text{cm})^3 \cdot 3\ \text{cm}}{36} + (1\ \text{cm} - 2{,}83\ \text{cm})^2 \cdot 4{,}5\ \text{cm}^2\right]$$
$$= \left[52{,}08\ \text{cm}^4 + 2{,}72\ \text{cm}^4\right] - \left[2{,}25\ \text{cm}^4 + 15{,}07\ \text{cm}^4\right] = 54{,}80\ \text{cm}^4 - 17{,}32\ \text{cm}^4 = 37{,}48\ \text{cm}^4$$

$$J_{xoyo} = \left[0 + (2{,}5\ cm - 2{,}83\ cm)\cdot(2{,}5\ cm - 2{,}83\ cm)\cdot 25\ cm^2\right] - \left[-\frac{(3\ cm)^2\cdot(3\ cm)^2}{72} + (1\ cm - 2{,}83\ cm)\cdot(1\ cm - 2{,}83\ cm)\cdot 4{,}5\ cm^2\right]$$

$$= 2{,}72\ cm^4 - 13{,}95\ cm^4 = -11{,}23\ cm^4$$

$$tg\ 2\alpha_o = \frac{-2\cdot J_{xoyo}}{J_{xo} - J_{yo}}$$

$$tg\ 2\alpha_o = \frac{(-2)\cdot(-11{,}23\ cm^4)}{37{,}48\ cm^4 - 37{,}48\ cm^4} = \frac{22{,}46\ cm^4}{0}$$

$$2\alpha_o = 90^\circ \quad /:2$$

$$\alpha_o = 45^\circ$$

$$J_{max} = \frac{J_{xo} + J_{yo}}{2} + \frac{1}{2}\cdot\sqrt{(J_{xo} - J_{yo})^2 + 4\cdot(J_{xoyo})^2}$$

$$J_{max} = \frac{37{,}48\ cm^4 + 37{,}48\ cm^4}{2} + \frac{1}{2}\cdot\sqrt{(37{,}48\ cm^4 - 37{,}48\ cm^4)^2 + 4\cdot(-11{,}23\ cm^4)^2}$$

$$= 37{,}48\ cm^4 + 11{,}23\ cm^4 = 48{,}71\ cm^4$$

$\underline{\underline{J_{max} = 48{,}71\ cm^4}}$ ($x_{gł}$)

$$J_{min} = \frac{J_{xo} + J_{yo}}{2} - \frac{1}{2}\cdot\sqrt{(J_{xo} - J_{yo})^2 + 4\cdot(J_{xoyo})^2}$$

$$J_{min} = \frac{37{,}48\ cm^4 + 37{,}48\ cm^4}{2} - \frac{1}{2}\cdot\sqrt{(37{,}48\ cm^4 - 37{,}48\ cm^4)^2 + 4\cdot(-11{,}23\ cm^4)^2}$$

$$= 37{,}48\ cm^4 - 11{,}23\ cm^4 = 26{,}25\ cm^4$$

$\underline{\underline{J_{min} = 26{,}25\ cm^4}}$ ($y_{gł}$)

<u>Sprawdzenie:</u>

1) $J_{xo} + J_{yo} = J_{max} + J_{min}$

$$37{,}48\ cm^4 + 37{,}48\ cm^4 = 48{,}71\ cm^4 + 26{,}25\ cm^4$$

$$74{,}96\ cm^4 = 74{,}96\ cm^4$$

$\underline{\underline{L = P}}$

Sprawdzono:	.... (data)	.... (podpis)

2) $J_{xgł} J_{ygł} = 0$

$$J_{xgł} J_{ygł} = \frac{J_{xo} - J_{yo}}{2} \cdot \sin 2\alpha_o + J_{xoyo} \cdot \cos 2\alpha_o$$

$$J_{xgł} J_{ygł} = \frac{37{,}48\ cm^4 - 37{,}48\ cm^4}{2} \cdot \sin 90^\circ + \left(-11{,}23\ cm^4\right) \cdot \cos 90^\circ = 0 \cdot 1 - 11{,}23\ cm^4 \cdot 0 = 0$$

Dewiacja:

$$i_{xgł} = \sqrt{\frac{J_{max,\,min}}{A}} = \sqrt{\frac{J_{max}}{A}} = \sqrt{\frac{48{,}71\ cm^4}{20{,}5\ cm^4}} = 1{,}54\ cm$$

$$i_{ygł} = \sqrt{\frac{J_{max,\,min}}{A}} = \sqrt{\frac{J_{min}}{A}} = \sqrt{\frac{26{,}25\ cm^4}{20{,}5\ cm^4}} = 1{,}13\ cm$$

$J_{max} = J_m$

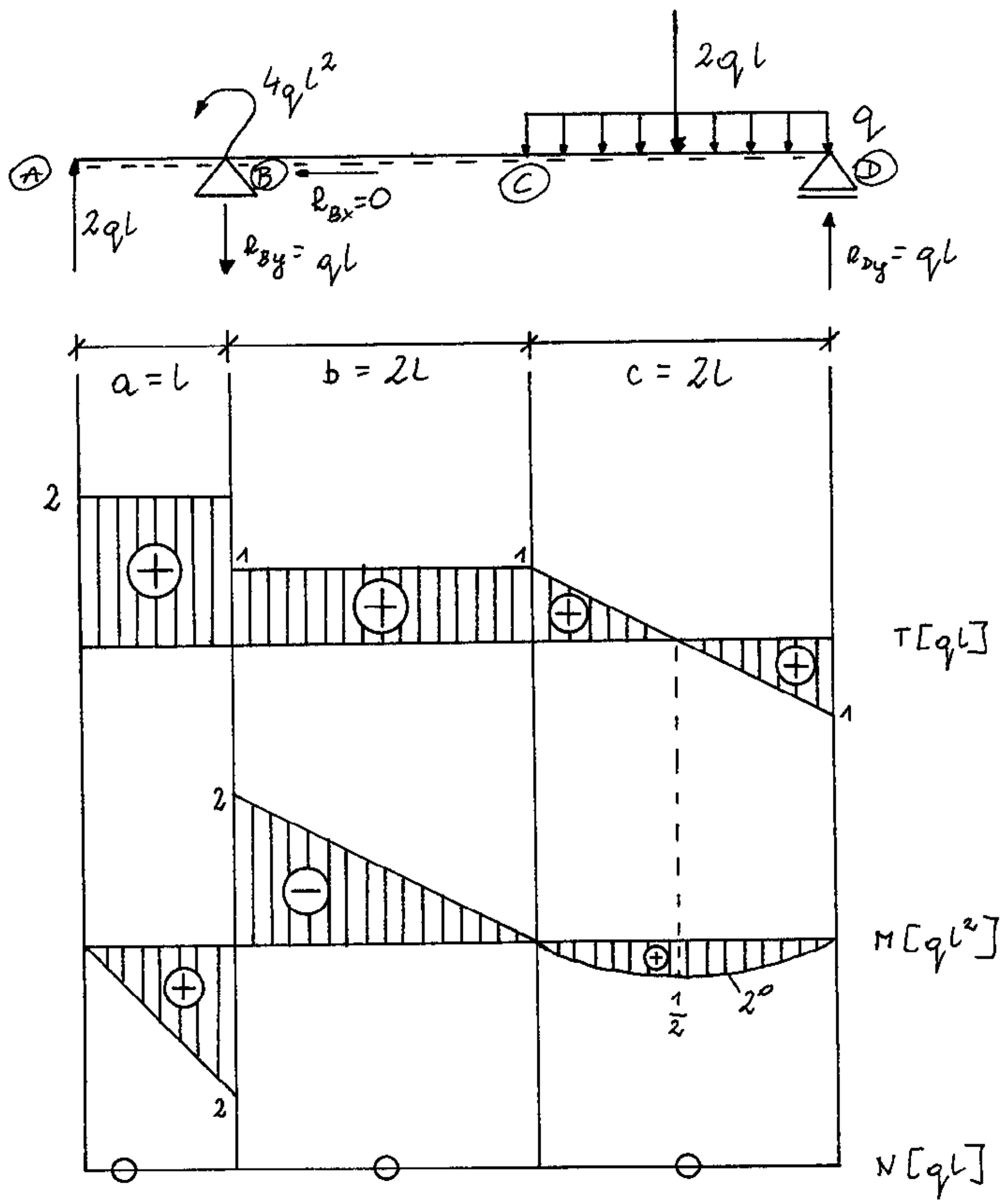
$$J_m = \frac{J_{xo} + J_{yo}}{2} + \frac{J_{xo} - J_{yo}}{2} \cdot \cos 2\alpha_o - J_{xoyo} \cdot \sin 2\alpha_o$$

$$J_m = \frac{37{,}48\ cm^4 + 37{,}48\ cm^4}{2} + \frac{37{,}48\ cm^4 - 37{,}48\ cm^4}{2} \cdot \cos 90^\circ - \left(-11{,}23\ cm^4\right) \cdot \sin 90^\circ$$
$$= 37{,}48\ cm^4 + 0 \cdot 0 + 11{,}23\ cm^4 \cdot 1 = 48{,}71\ cm^4$$

$J_{max} = J_n$

$$J_n = \frac{J_{xo} + J_{yo}}{2} - \frac{J_{xo} - J_{yo}}{2} \cdot \cos 2\alpha_o + J_{xoyo} \cdot \sin 2\alpha_o$$

$$J_n = \frac{37{,}48\ cm^4 + 37{,}48\ cm^4}{2} - \frac{37{,}48\ cm^4 - 37{,}48\ cm^4}{2} \cdot \cos 90^\circ + \left(-11{,}23\ cm^4\right) \cdot \sin 90^\circ$$
$$= 37{,}48\ cm^4 - 0 \cdot 0 - 11{,}23\ cm^4 \cdot 1 = 26{,}25\ cm^4$$



$R = 175$ MPa

$R_t = 105$ MPa

$q = 1$ kN/m

$l = 2$ m

Sprawdzono:	.... (data)	.... (podpis)

$\sigma_{max} \leq R$

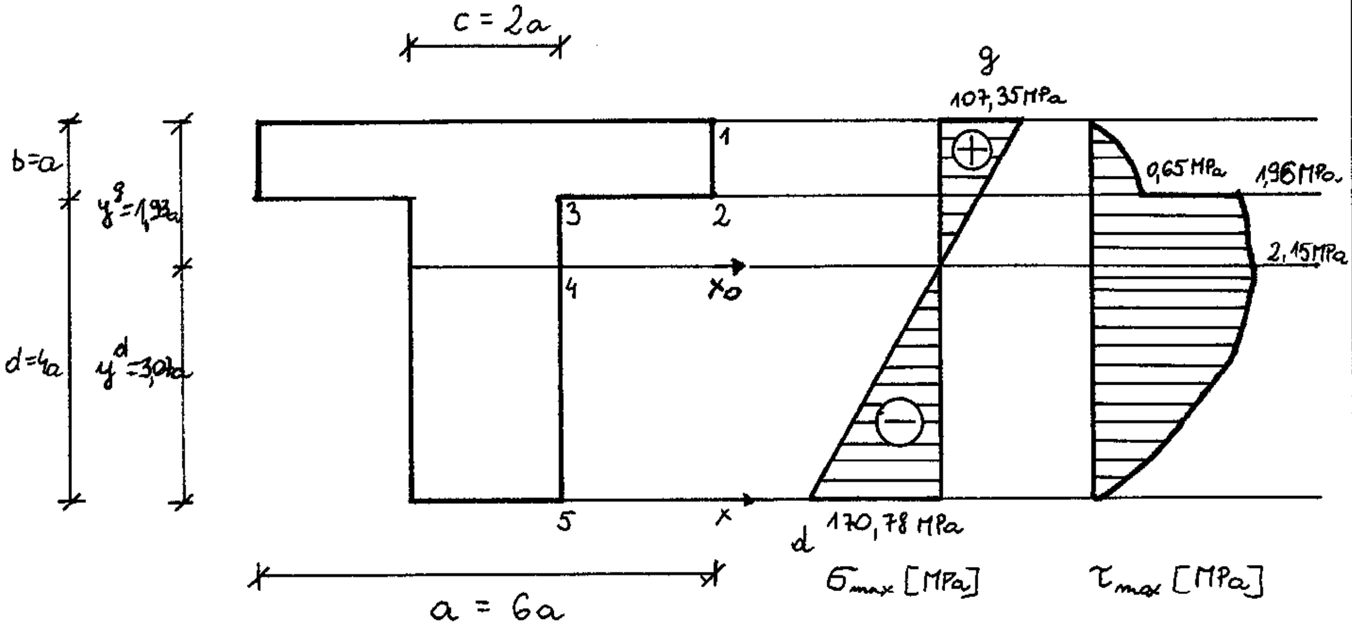
$$\sigma_{max} = \frac{M_{max}}{W_{x\,min}}$$

$\tau_{max} \leq R_t$

$$\tau_{max} = \frac{T_{max} \cdot S_x}{b \cdot J_{xo}}$$

$$T_{max} = 2 \cdot q \cdot l = 2 \cdot 1\,kN/m \cdot 2\,m = 4\,kN$$

$$M_{max} = 2 \cdot q \cdot l^2 = 2 \cdot 1\,kN/m \cdot (2\,m)^2 = 8\,kNm$$



$$A = 4\,a \cdot 2\,cm + 6\,a \cdot a = 8\,a^2 + 6\,a^2 = 14\,a^2$$

$$S_x = 8\,a^2 \cdot 2\,a + 6\,a^2 \cdot 4{,}5\,a = 16\,a^3 + 27\,a^3 = 43\,a^3$$

$$y_o = \frac{S_x}{A} = \frac{43\,a^3}{14\,a^2} = 3{,}07\,a$$

$$J_{xo} = \left[\frac{2\,a \cdot (4\,a)^3}{12} + (2\,a - 3{,}07\,a)^2 \cdot 8\,a^2\right] + \left[\frac{6\,a \cdot (a)^3}{12} + (4{,}5\,a - 3{,}07\,a)^2 \cdot 6\,a^2\right] =$$

$$= [19{,}83\,a^4 + 12{,}77\,a^4] = 32{,}6\,a^4$$

$$W_x^{\,g} = \frac{J_{xo}}{y^g} = \frac{32{,}6\ a^4}{1{,}93\ a} = 16{,}89\ a^3$$

$$W_x^{\,d} = \frac{J_{xo}}{y^d} = \frac{32{,}6\ a^4}{3{,}07\ a} = 10{,}62\ a^3 = W_{x\,min}$$

$$\sigma_{max} \le R$$

$$\frac{M_{max}}{W_{x\,min}} \le R$$

$$\frac{8\ kNm}{10{,}62\ a^3} \le 175\ MPa$$

$$\frac{800\ kNcm}{10{,}62\ a^3} \le 17{,}5\ kN/cm^2$$

$$10{,}62\ a^3 \ge \frac{800\ kNcm}{17{,}5\ kN/cm^2}$$

$$a^3 \ge 4{,}30\ cm^3$$

$$\underline{\underline{a \ge 1{,}63\ cm}}$$

$$\tau_{max} \le R_t$$

$$\frac{T_{max} \cdot S_x}{b \cdot J_{xo}} \le R_t$$

$$\frac{4\ kN \cdot (2\ a \cdot 3{,}07\ a \cdot 1{,}535\ a)}{2\ a \cdot 32{,}6\ a^4} \le 105\ MPa$$

$$\frac{4\ kN \cdot 9{,}4349\ a^3}{65{,}2\ a^5} \le 10{,}5\ kN/cm^2$$

$$\frac{65{,}2\ a^5}{9{,}4349\ a^3} \ge \frac{4\ kN}{10{,}5\ kN/cm^2}$$

$$a^2 \ge 0{,}055\ cm^2$$

$$\underline{\underline{a \ge 0{,}23\ cm}}$$

Sprawdzono:	.... (data)	.... (podpis)

Przyjmuję a = 1,64 cm

$$\sigma^{g}_{max} = \frac{M_{max}}{W_x^{g}} = \frac{8\ kNm}{16{,}89\ a^3} = \frac{800\ kNcm}{16{,}89 \cdot (1{,}64\ cm)^3} = 10{,}738\ kN/cm^2 = 107{,}38\ MPa < R$$

$$\sigma^{d}_{max} = \frac{M_{max}}{W_x^{d}} = \frac{8\ kNm}{10{,}62\ a^3} = \frac{800\ kNcm}{10{,}62 \cdot (1{,}64\ cm)^3} = 17{,}078\ kN/cm^2 = 170{,}78\ MPa < R$$

$$\tau_1 = 0$$

$$\tau_2 = \frac{4\ kN \cdot (6\ a^2 \cdot 1{,}43\ a)}{6\ a \cdot 32{,}6\ a^4} = 0{,}065\ kN/cm^2 = 0{,}65\ MPa < R_t$$

$$\tau_3 = \frac{4\ kN \cdot (6\ a^2 \cdot 1{,}43\ a)}{2\ a \cdot 32{,}6\ a^4} = 0{,}196\ kN/cm^2 = 1{,}96\ MPa < R_t$$

$$\tau_4 = \tau_{max} = \frac{4\ kN \cdot (3{,}07\ a \cdot 2\ a \cdot 1{,}535\ a)}{2\ a \cdot 32{,}6\ a^4} = 0{,}215\ kN/cm^2 = 2{,}15\ MPa < R_t$$

$$\tau_5 = 0$$