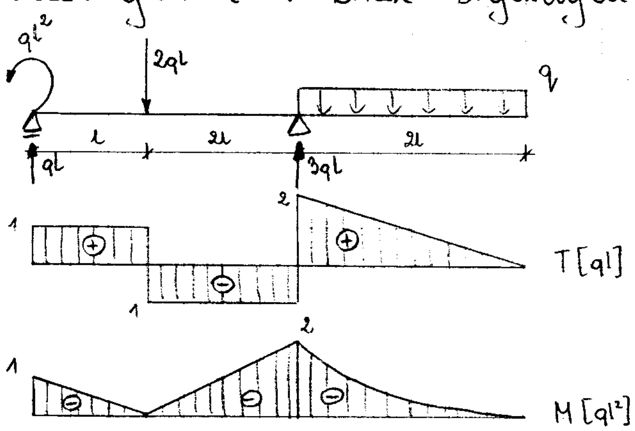
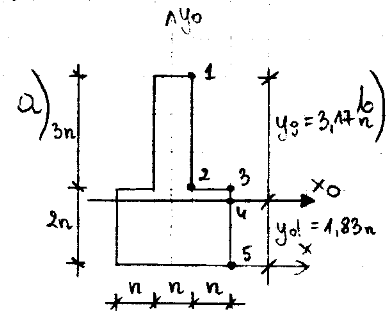
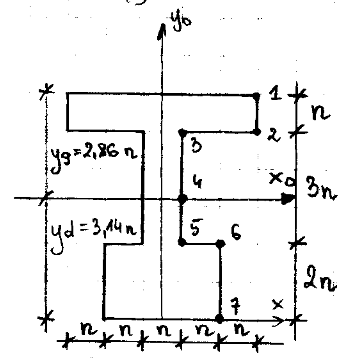
Dla podanej belki zaprojektować przekrój poprzeczny, przyjmując że dopuszczalne naprężenia normalne $R = 175\ MPa$, naprężenia styczne $R_t = 105\ MPa$, wartości $q = 2\ kN/m$, $l = 2\ m$. Sporządzić wykresy ekstremalnych naprężeń normalnych (σ) oraz stycznych (τ).







$$T_{max} = 2ql = 2kN/m \cdot 2m \cdot 2 = 8\ kN$$

$$M_{max} = 2ql^2 = 2 \cdot 2kN/m \cdot (2m)^2 = 16\ kNm$$

$$\sigma_{max} = \frac{M_{max}}{W_{x\,min}} \le R \qquad \tau_{max} = \frac{T_{max} \cdot S_x}{b \cdot J_{x_0}} \le R_t$$

a) $A = 6n^2 + 3n^2 = 9n^2$ $\qquad y_0 = \frac{S_x}{A} = \frac{16{,}5n^3}{9n^2} = 1{,}83n$

$$S_x = 6n^2 \cdot n + 3n^2 \cdot 3{,}5n = 16{,}5n^3$$

$$J_{x_0} = \left[\frac{3n \cdot (2n)^3}{12} + (n - 1{,}83n)^2 \cdot 6n^2\right] + \left[\frac{n \cdot (3n)^3}{12} + (3{,}5n - 1{,}83n)^2 \cdot 3n^2\right] = 16{,}75n^4$$

$$W_x^g = \frac{J_{x_0}}{y^g} = \frac{16{,}75n^4}{3{,}17n} = 5{,}28n^3 = W_{x\,min}$$

$$W_x^d = \frac{J_{x_0}}{y^d} = \frac{16{,}75n^4}{1{,}83n} = 9{,}15n^3$$

$$\sigma_{max} = \frac{M_{max}}{W_{x\,min}} \le R \Rightarrow \frac{16kNm}{5{,}28n^3} \le 175\ MPa \Rightarrow \frac{1600\ kNcm}{5{,}28n^3} \le 17{,}5\ \frac{kN}{cm^2}$$

$$5{,}28n^3 \ge \frac{1600\ kNcm}{17{,}5\ kN/cm^2} \Rightarrow n^3 \ge 17{,}32\ cm^3 \Rightarrow \underline{n \ge 2{,}59\ cm}$$

$$\tau_{max} = \frac{T_{max} \cdot S_x}{b \cdot J_{x_0}} \le R_t \Rightarrow \tau_{max} = \tau_2 \Rightarrow \frac{8kN \cdot (3n \cdot n \cdot 1{,}67n)}{n \cdot 16{,}75n^4} \le 105\ MPa$$

$$\frac{40{,}08\ kN \cdot n^{3}}{16{,}75n^{5}} \le 10{,}5\ kN/cm^2 \Rightarrow 16{,}75n^2 \ge \frac{40{,}08\ kN}{10{,}5\ kN/cm^2} \Rightarrow n^2 \ge 0{,}23\ cm^2 \Rightarrow$$

$$\underline{n \ge 0{,}48\ cm}$$

Przyjmuję $n = 2{,}6\ cm$

$$\sigma^g_{max} = \frac{M_{max}}{W_x^g} = \frac{16kNm}{5{,}28n^3} = \frac{1600\ kNcm}{5{,}28 \cdot (2{,}6cm)^3} = 17{,}241\ kN/cm^2 = 172{,}41\ MPa < R$$

$$\sigma^d_{max} = \frac{M_{max}}{W_x^d} = \frac{16kNm}{9{,}15n^3} = \frac{1600\ kNcm}{9{,}15 \cdot (2{,}6cm)^3} = 9{,}949\ kN/cm^2 = 99{,}49\ MPa < R$$

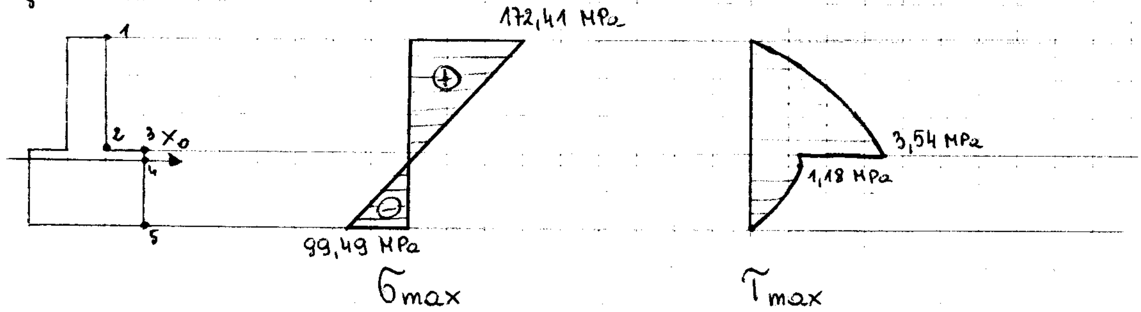
$$\tau_1 = 0$$

$$\tau_2 = \tau_{max} = \frac{8kN \cdot (3n \cdot n \cdot 1{,}67n)}{n \cdot 16{,}75n^4} = \frac{40{,}08\ kN}{16{,}75n^2} = 0{,}354\ kN/cm^2 = 3{,}54\ MPa < R_t$$

$$\tau_3 = \frac{8kN \cdot (3n \cdot n \cdot 1{,}67n)}{3n \cdot 16{,}75n^4} = \frac{40{,}08\ kN}{50{,}25n^2} = 0{,}118\ kN/cm^2 = 1{,}18\ MPa < R_t$$

$$\tau_4 = \frac{8kN \cdot (1{,}83n \cdot 3n \cdot 0{,}915n)}{3n \cdot 16{,}75 n^4} = \frac{40{,}1868\ kN}{50{,}25\ n^2} = 0{,}118\ kN/cm^2 = 1{,}18\ MPa < R_t$$

$$\tau_5 = 0$$



b) $A = 6n^2 + 3n^2 + 5n^2 = 14n^2$ $\qquad y_0 = \frac{S_x}{A} = \frac{44n^3}{14n^2} = 3{,}14n$

$$S_x = 6n^2 \cdot n + 3n^2 \cdot 3{,}5n + 5n^2 \cdot 5{,}5n = 44n^3$$

$$J_{x_0} = \left[\frac{3n\cdot(2n)^3}{12} + (n - 3{,}14n)^2 \cdot 6n^2\right] + \left[\frac{n\cdot(3n)^3}{12} + (3{,}5n - 3{,}14n)^2 \cdot 3n^2\right] + \left[\frac{5n\cdot(n)^3}{12} + (5{,}5n - 3{,}14n)^2 \cdot 5n^2\right] = 60{,}38n^4$$

$$W_x^g = \frac{J_{x_0}}{y_g} = \frac{60{,}38n^4}{2{,}86n} = 21{,}11n^3$$

$$W_x^d = \frac{J_{x_0}}{y_d} = \frac{60{,}38n^4}{3{,}14n} = 19{,}23n^3 = W_{x\,min}$$

$$\frac{M_{max}}{W_x^{min}} \le R \Rightarrow \frac{16\ kNm}{19{,}23n^3} \le 175\ MPa \Rightarrow 19{,}23n^3 \ge \frac{1600\ kNcm}{17{,}5\ kN/cm^2} \Rightarrow n^3 \ge 4{,}75\ cm^3$$

$n \ge 1{,}68\ cm$

$$\frac{T_{max} \cdot S_x}{b \cdot J_{x_0}} \le R_t \qquad \tau_{max} = \tau_4 \qquad \frac{8kN \cdot (5n \cdot n \cdot 2{,}36n + n \cdot 1{,}86n \cdot 0{,}93n)}{n \cdot 60{,}38n^4} \le 105\ MPa \Rightarrow$$

$$\frac{108{,}2384\ n^3 \cdot kN}{60{,}38\ n^5} \le 10{,}5\ kN/cm^2 \Rightarrow 60{,}38n^2 \ge \frac{108{,}2384\ kN}{10{,}5\ kN/cm^2} \Rightarrow n^2 \ge 0{,}17\ cm^2 \Rightarrow$$

$n \ge 0{,}41\ cm$

Przyjmuję $n = 1{,}7\ cm$

$$\sigma_{max}^g = \frac{M_{max}}{W_x^g} = \frac{16\ kNm}{21{,}11n^3} = \frac{1600\ kNcm}{21{,}11 \cdot (1{,}7cm)^3} = 15{,}427\ kN/cm^2 = 154{,}27\ MPa < R$$

$$\sigma_{max}^d = \frac{M_{max}}{W_x^d} = \frac{16\ kNm}{19{,}23n^3} = \frac{1600\ kNcm}{19{,}23 \cdot (1{,}7cm)^3} = 16{,}935\ kN/cm^2 = 169{,}35\ MPa < R$$

$$\tau_1 = 0$$

$$\tau_2 = \frac{8kN \cdot (5n \cdot n \cdot 2{,}36n)}{5n \cdot 60{,}38n^4} = \frac{94{,}4\ kN}{301{,}9\ n^2} = 0{,}108\ kN/cm^2 = 1{,}08\ MPa < R_t$$

$$\tau_3 = \frac{8kN\ (5n \cdot n \cdot 2{,}86n)}{n \cdot 60{,}38n^4} = \frac{94{,}4\ kN}{60{,}38\ n^2} = 0{,}541\ kN/cm^2 = 5{,}41\ MPa < R_t$$

$$\tau_4 = \tau_{max} = \frac{8kN \cdot (5n \cdot n \cdot 2{,}36n + 1{,}86 \cdot n \cdot 0{,}93n)}{n \cdot 60{,}38n^4} = \frac{108{,}2384\ kN}{60{,}38\ n^2} = 0{,}62\ kN/cm^2 = 6{,}2\ MPa$$

$$\tau_5 = \frac{8kN \cdot (3n \cdot 2n \cdot 2{,}14n)}{n \cdot 60{,}38n^4} = \frac{102{,}72\ kN}{60{,}38\ n^2} = 0{,}589\ kN/cm^2 = 5{,}89\ MPa < R_t$$

$$\tau_6 = \frac{8kN \cdot (3n \cdot 2n \cdot 2{,}14n)}{3n \cdot 60{,}38\ n^4} = \frac{102{,}72\ kN}{181{,}14\ n^2} = 0{,}196\ kN/cm^2 = 1{,}96\ MPa < R_t$$

$$\tau_7 = 0$$