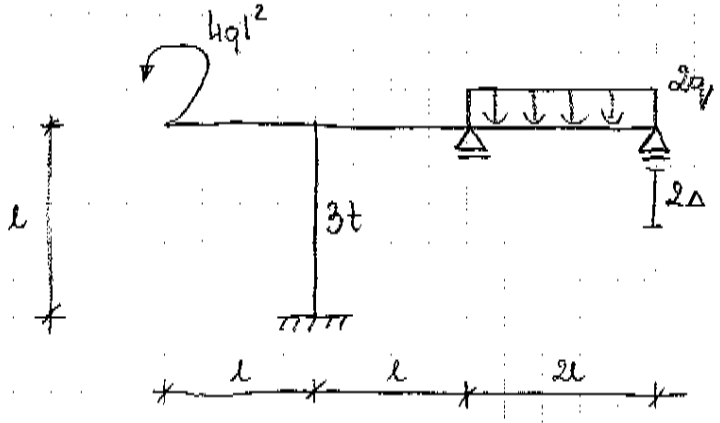
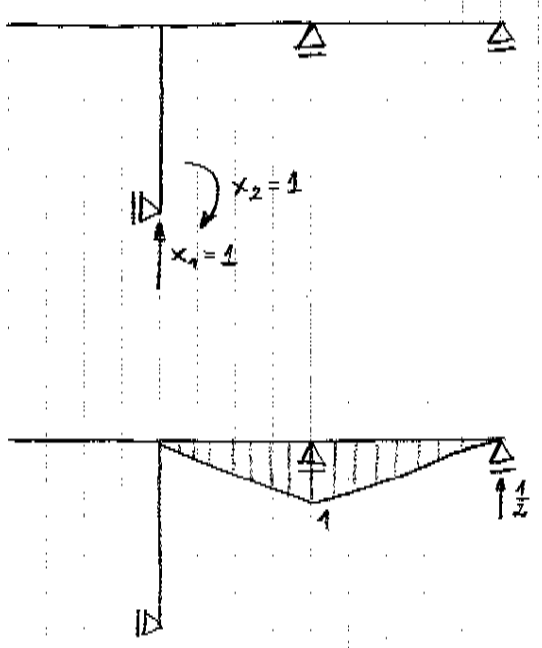
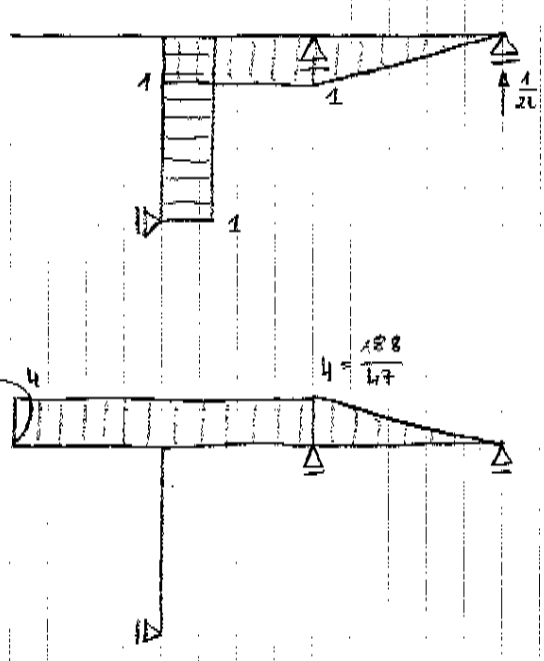


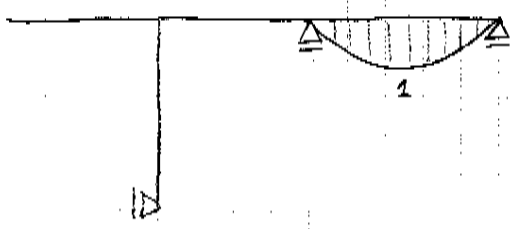


M_1 [l]

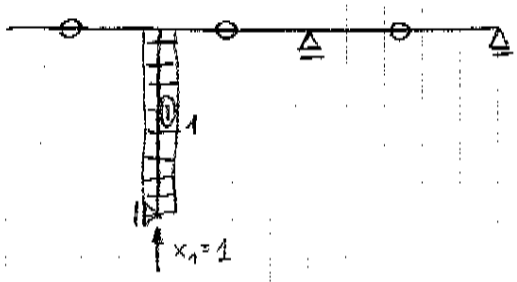


M_2 [-]

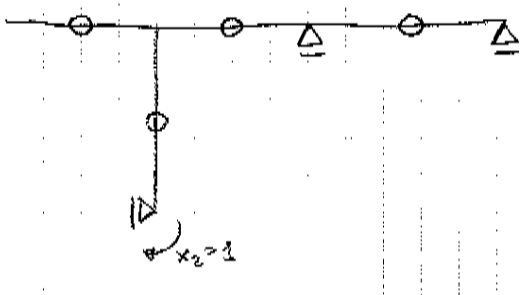
M_{PM} [ql^2]



M_{pq} $[ql^2]$



N_1 $[-]$



N_2 $\left[\frac{1}{l}\right]$

$$\delta_{11} = \frac{1}{EJ}\left[\frac{1}{2}\cdot l\cdot l\cdot\frac{2}{3}l + \frac{1}{2}\cdot l\cdot 2l\cdot\frac{2}{3}l\right] = \frac{l^3}{EJ}$$

$$\delta_{12} = \frac{1}{EJ}\left[1\cdot l\cdot\left(\frac{1}{2}\cdot l\right) + \frac{1}{2}\cdot 1\cdot 2l\cdot\left(\frac{2}{3}l\right)\right] = \frac{7}{6}\frac{l^2}{EJ}$$

$$\delta_{21} = \frac{1}{EJ}\left[\frac{1}{2}\cdot l\cdot l\cdot 1 + \frac{1}{2}\cdot l\cdot 2l\cdot\left(\frac{2}{3}\cdot 1\right)\right] = \frac{7}{6}\frac{l^2}{EJ}$$

$$\delta_{22} = \frac{1}{EJ}\left[1\cdot l\cdot 1 + 1\cdot l\cdot 1 + \frac{1}{2}\cdot 2l\cdot 1\cdot\frac{2}{3}\right] = \frac{8}{3}\frac{l}{EJ}$$

$$\Delta_{1M} = \frac{1}{EJ}\left[\frac{1}{2}\cdot 4ql^2\cdot l\cdot\left(-\frac{1}{2}l\right) + \frac{1}{2}\cdot 4ql^2\cdot 2l\cdot\left(-\frac{2}{3}\cdot l\right)\right] = -\frac{14}{3}\frac{ql^4}{EJ}$$

$$\Delta_{2M} = \frac{1}{EJ}\left[4ql^2\cdot l\cdot(-1) + \frac{1}{2}\cdot 4ql^2\cdot 2l\cdot\left(-\frac{2}{3}\right)\right] = -\frac{20}{3}\frac{ql^3}{EJ}$$

$$\Delta_{1q} = \frac{1}{EJ}\left[\frac{2}{3}\cdot 1ql^2\cdot 2l\cdot\left(\frac{1}{2}l\right)\right] = \frac{2}{3}\frac{ql^4}{EJ}$$

$$\Delta_{2q} = \frac{1}{EJ}\left[\frac{2}{3}\cdot 1ql^2\cdot 2l\cdot\left(\frac{1}{2}\right)\right] = \frac{2}{3}\frac{ql^3}{EJ}$$

$$\Delta_{1t} = -1\cdot l\,\alpha_t\cdot 3t = -3\,\alpha_t\cdot l\cdot t$$

$$\Delta_{2t} = 0$$

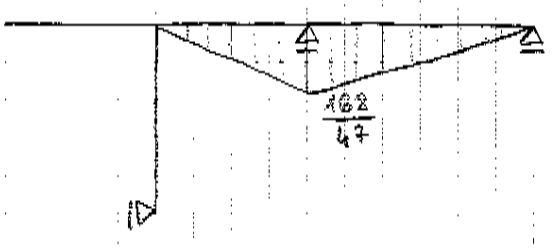
$$\Delta_{1\Delta} = -\left(-\frac{1}{2}\cdot 2\Delta\right) = \Delta$$

$$\Delta_{2\Delta} = -\left(-\frac{1}{2l}\cdot 2\Delta\right) = \frac{\Delta}{l}$$

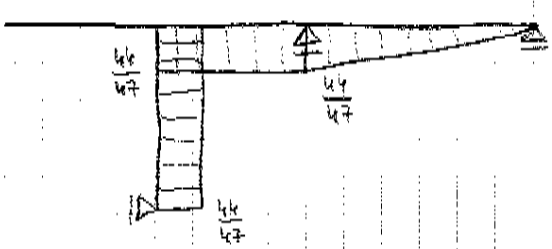
Obciążenie momentem skupionym

$$\begin{cases} 1\,\frac{l^3}{EJ}\cdot X_1 + \frac{7}{6}\,\frac{l^2}{EJ}\cdot X_2 - \frac{14}{3}\,\frac{ql^4}{EJ} = 0 \\ \frac{7}{6}\,\frac{l^2}{EJ}\cdot X_1 + \frac{8}{3}\,\frac{l}{EJ}\cdot X_2 - \frac{20}{3}\,\frac{ql^3}{EJ} = 0 \end{cases}$$

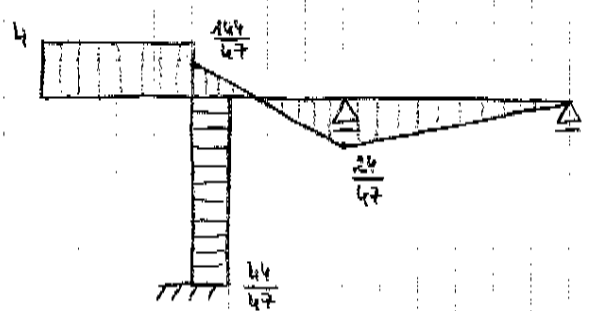
$$\begin{cases} X_1 = \frac{168}{47}\,ql \\ X_2 = \frac{44}{47}\,ql^2 \end{cases}$$



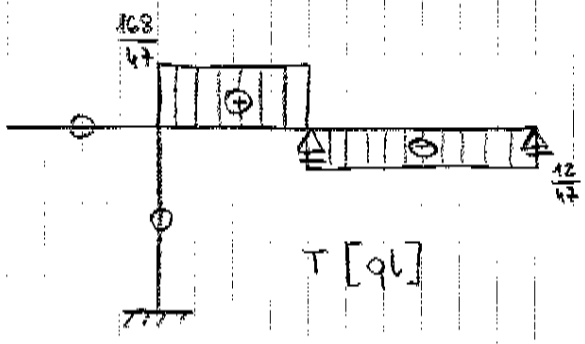
$M_1 \cdot X_1$ $[ql^2]$



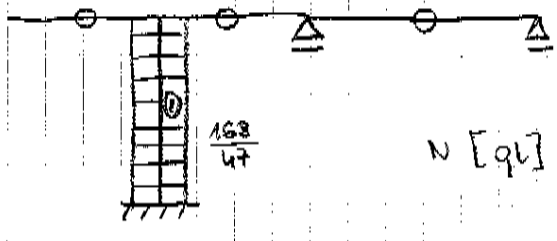
$M_2 \cdot X_2$ $[ql^2]$



M $[ql^2]$



T $[ql]$

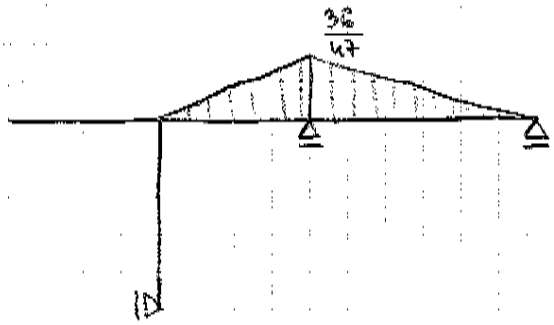


N $[ql]$

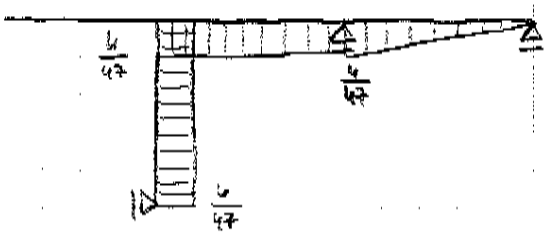
Obciążenie ciągłe

$$\begin{cases} 1\,\frac{l^3}{EJ}\cdot X_1 + \frac{7}{6}\,\frac{l^2}{EJ} + \frac{2}{3}\,\frac{ql^4}{EJ} = 0 \\ \frac{7}{6}\,\frac{l^2}{EJ}\cdot X_1 + \frac{8}{3}\,\frac{l}{EJ} + \frac{2}{3}\,\frac{ql^3}{EJ} = 0 \end{cases}$$

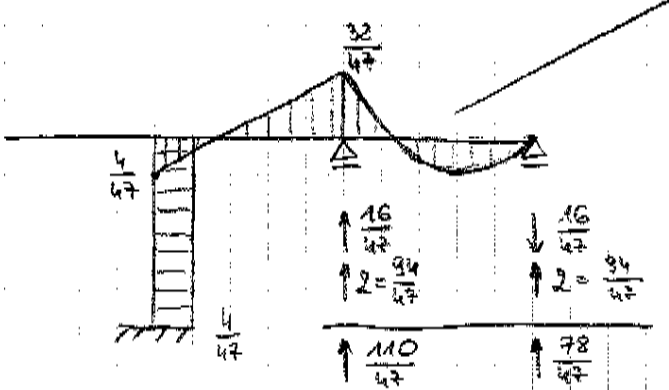
$$\begin{cases} X_1 = -\frac{36}{47}\,ql \\ X_2 = \frac{4}{47}\,ql^2 \end{cases}$$

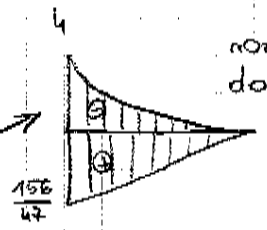


$M_1 \cdot X_1$ [ql^2]



$M_2 \cdot X_2$ [ql^2]

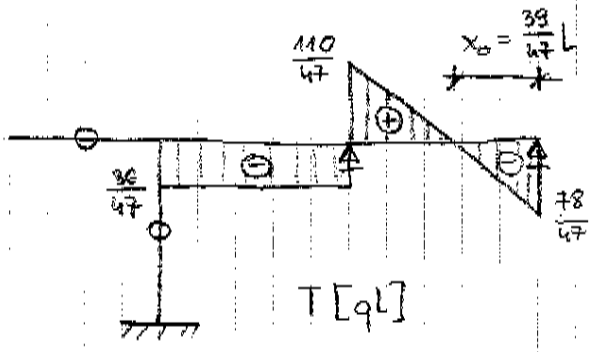




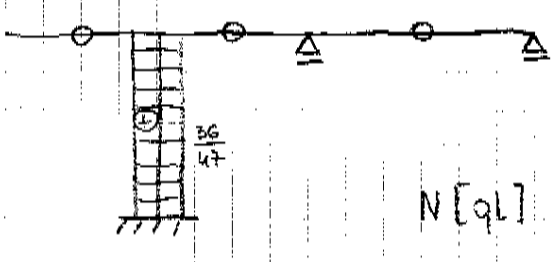
rozłożony wykres do sprawdzenia

M [ql^2]

$$M_{max} = \frac{78}{47}\,ql\cdot\frac{39}{47}\,l - 2q\cdot\frac{39}{47}\,l\cdot\frac{1}{2}\cdot\frac{39}{47}\,l = \frac{1521}{2209}\,ql^2$$



T [ql]

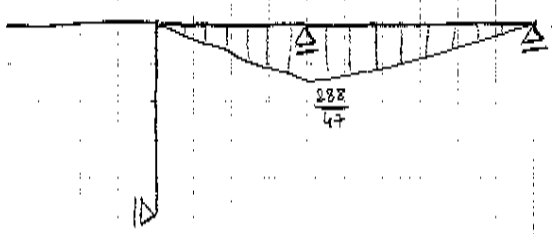


N [ql]

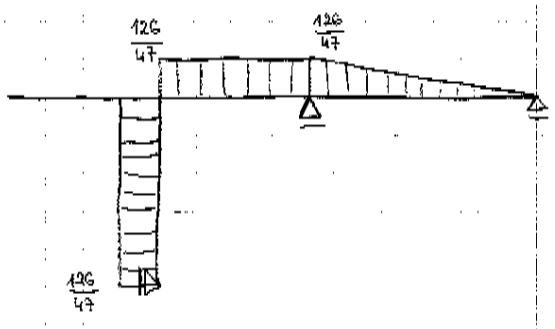
Obciążenie termiczne

$$\begin{cases} 1\frac{l^3}{EJ}\,X_1 + \frac{7}{6}\frac{l^2}{EJ}\,X_2 - 3\alpha_t \cdot l \cdot t = 0 \\ \frac{7}{6}\frac{l^2}{EJ}\,X_1 + \frac{8}{3}\frac{l}{EJ}\,X_2 = 0 \end{cases}$$

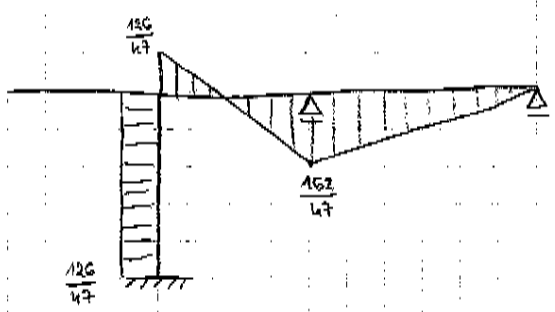
$$\begin{cases} X_1 = \frac{288}{47}\,\frac{EJ\,\alpha_t\,t}{l^2} \\ X_2 = -\frac{126}{47}\,\frac{EJ\cdot\alpha_t\cdot t}{l} \end{cases}$$



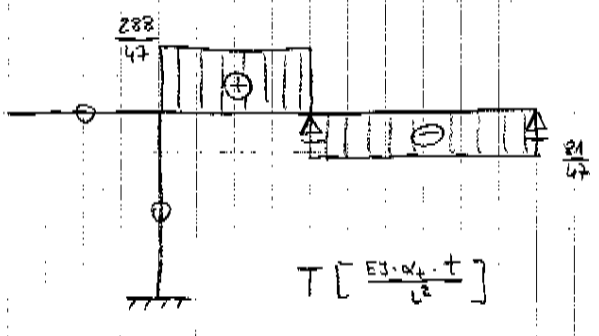
$M_1 \cdot X_1 \left[\frac{EJ\cdot\alpha_t\,t}{l}\right]$



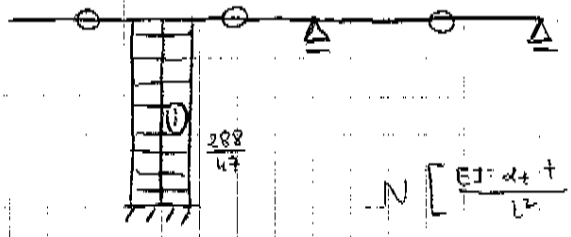
$M_2 \cdot X_2 \left[\frac{EJ\cdot\alpha_t\cdot t}{l}\right]$



$M \left[\frac{EJ\cdot\alpha_t\,t}{l}\right]$

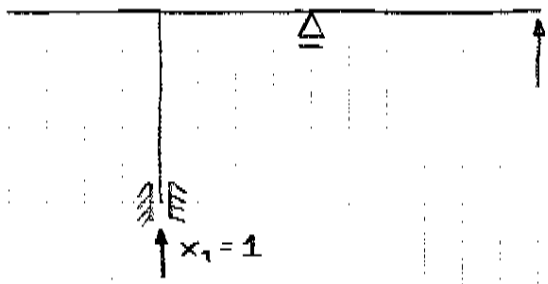


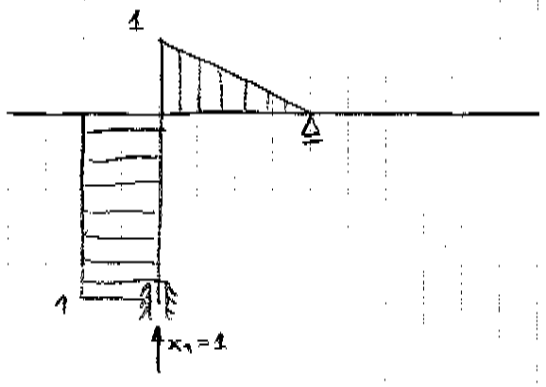
$T \left[\frac{EJ\cdot\alpha_t\cdot t}{l^2}\right]$



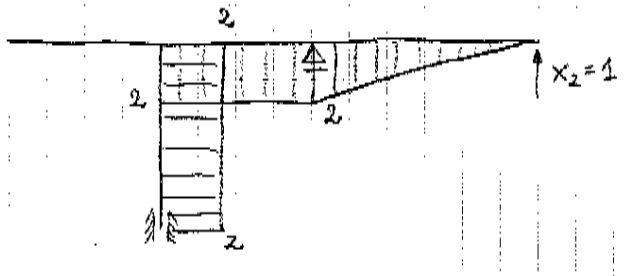
$N \left[\frac{EJ\cdot\alpha_t\,t}{l^2}\right]$

Sprawdzenie





M_1^1 [l]



M_2^1 [l]

1) Obciążenie momentem skupionym

$$f_1 = \frac{1}{EJ}\left[\frac{1}{2}\cdot\frac{144}{47}ql^2\cdot l\cdot\frac{2}{3}l + \frac{1}{2}\cdot\frac{24}{47}ql^2\cdot l\cdot\left(-\frac{1}{3}l\right) + \frac{44}{47}ql^2\cdot l\cdot(-l)\right] = 0$$

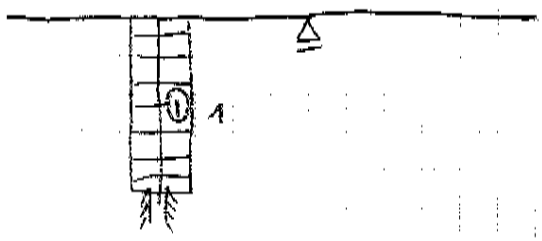
$$f_2 = \frac{1}{EJ}\left[\frac{1}{2}\cdot\frac{144}{47}ql^2\cdot l\cdot(-2l) + \frac{1}{2}\cdot\frac{24}{47}ql^2\cdot l\cdot(2l) + \frac{1}{2}\cdot\frac{24}{47}ql^2\cdot 2l\cdot\left(\frac{2}{3}\cdot 2l\right) + \frac{44}{47}ql^2\cdot l\cdot 2l\right] =$$
$$= 0$$

2) Obciążenie ciągłe

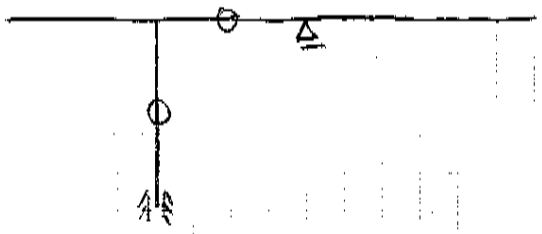
$$f_1 = \frac{1}{EJ}\left[\frac{1}{2}\cdot\frac{4}{47}ql^2\cdot l\cdot\left(-\frac{2}{3}l\right) + \frac{1}{2}\cdot\frac{32}{47}ql^2\cdot l\cdot\frac{1}{3}l + \frac{4}{47}ql^2\cdot l\cdot(-l)\right] = 0$$

$$f_2 = \frac{1}{EJ}\left[\frac{1}{2}\cdot\frac{4}{47}ql^2\cdot l\cdot 2l + \frac{1}{2}\cdot\frac{32}{47}ql^2\cdot l\cdot(-2l) + \frac{4}{47}ql^2\cdot l\cdot 2l + \frac{1}{3}\cdot 4ql^2\cdot 2l\cdot\left(-\frac{3}{4}\cdot 2l\right) +\right.$$
$$\left. + \frac{1}{2}\cdot\frac{156}{47}ql^2\cdot 2l\cdot\left(\frac{2}{3}\cdot 2l\right)\right] = 0$$

3) Obciążenie termiczne



N_1' [-]



N_2' [-]

$$f_1 = \frac{1}{EJ}\left[\frac{1}{2}\cdot\frac{12G}{47}\frac{EJ\cdot\alpha_t\cdot t}{l}\cdot l\,\frac{2}{3}l + \frac{1}{2}\cdot\frac{16Z}{47}\frac{EJ\cdot\alpha_t\cdot t}{l}\cdot l\cdot\left(-\frac{1}{3}l\right) + \frac{12G}{47}\frac{EJ\cdot\alpha_t\cdot t}{l}\cdot l\cdot l\right] - 1\cdot l\cdot\alpha_t\cdot 3t =$$

$$= 3t\alpha_t\cdot l - 3t\alpha_t\cdot l = 0$$

$$f_2 = \frac{1}{EJ}\left[\frac{1}{2}\cdot\frac{12G}{47}\cdot\frac{EJ\cdot\alpha_t\cdot t}{l}\cdot l\cdot(-2l) + \frac{1}{2}\cdot\frac{16Z}{47}\cdot\frac{EJ\cdot\alpha_t\cdot t}{l}\cdot l\cdot 2l + \frac{1}{2}\cdot\frac{16Z}{47}\cdot\frac{EJ\cdot\alpha_t\cdot t}{l}\cdot 2l\cdot\left(\frac{2}{3}\cdot 2l\right) +\right.$$

$$\left. + \frac{12G}{47}\cdot\frac{EJ\cdot\alpha_t\cdot t}{l}\cdot l\cdot(-2l)\right] = 0$$

4) Obciążenie kinematyczne

$$f_1 = \frac{1}{EJ}\left[\frac{1}{2}\cdot\frac{G}{47}\cdot\frac{\Delta EJ}{l^2}\cdot l\cdot\left(-\frac{2}{3}l\right) + \frac{1}{2}\cdot\frac{48}{47}\cdot\frac{\Delta EJ}{l^2}\cdot l\cdot\frac{1}{3}l + \frac{G}{47}\cdot\frac{\Delta EJ}{l^2}\cdot l\cdot(-l)\right] = 0$$

$$f_2 = \frac{1}{EJ}\left[\frac{1}{2}\cdot\frac{G}{47}\cdot\frac{\Delta EJ}{l^2}\cdot l\cdot 2l + \frac{1}{2}\cdot\frac{48}{47}\cdot\frac{\Delta EJ}{l^2}\cdot l\cdot(-2l) + \frac{1}{2}\cdot\frac{48}{47}\cdot\frac{\Delta EJ}{l^2}\cdot 2l\cdot\left(-\frac{2}{3}\cdot 2l\right) + \frac{G}{47}\cdot\frac{\Delta EJ}{l^2}\cdot l\cdot 2l\right] -$$

$$-(-1\cdot 2\Delta) = -2\Delta + 2\Delta = 0$$